

Baryons and Pentaquarks in terms of Mesons

Ramesh Anishetty* and Santosh Kumar Kudtarkar†

The Institute of Mathematical Sciences, CIT-Campus, Taramani, Chennai, India

(Dated: February 2, 2008)

We investigate light meson bound state contributions to six quark Green's functions and establish that the latter has poles corresponding to baryon bound states. We estimate light baryon masses including the Roper resonance. Constituent quark model and the Parton model are natural consequences of this quantum field theoretic picture. We elaborate this analysis to pentaquarks and heptaquarks. An almost model independent prediction for the mass of θ pentaquark is 1.57GeV .

PACS numbers: 11.55.-m, 11.80.-m, 11.80.Jy, 12.38.-t, 12.39.Mk, 12.40.Yx, 14.20.-c

We look for a baryon pole in six quark Green's functions $\langle qq\bar{q}\bar{q}\bar{q}\bar{q} \rangle$ in QCD. Non-perturbatively this has many contributions, in particular let us concentrate on contributions due to quark-quark scattering. In a relativistic Quantum field theory like QCD this can be inferred from quark-antiquark scattering by crossing symmetry. We assume the color singlet s channel of $q\bar{q}$ scattering does have color singlet meson poles which are the observed pion, ρ, ω, K etc(Fig[1]). The wave-function ϕ_n of these mesons are realised as eigensolutions of the Bethe-Salpeter(BS) equation wherein a kernel is subsumed to exist from the underlying theory, QCD.

$$\langle q\bar{q} q\bar{q} \rangle = \sum_n \phi_n^* \frac{1}{s - M_n^2 + i\epsilon} \phi_n + \dots \quad (1)$$

where ϕ_n obeys the BS equation (Fig[1b]). M_n is the mass of the meson of type n .

$$\phi_n = S(p) \bar{S}(\bar{p}) \int K \phi_n \quad (2)$$

$S(p)$ and $\bar{S}(\bar{p})$ are the exact propagators for the quarks. The BS wave-function does have a spectral representation which follows from causality[1, 2]. It is given by

$$\phi_n = \int_0^1 dy \int_0^\infty \frac{d\beta}{yp^2 + (1-y)\bar{p}^2 - y(1-y)M_n^2 - \delta_n^2 - \beta + i\epsilon} \tilde{\phi}_n(y, \beta) \quad (3)$$

The spin angular momentum, flavour details are suppressed in the above equation. More precise definitions along with spin-orbit coupling details are presented in

[2] along with asymptotic behaviour of the BS wave-function which are deduced from the structure of the BS equation. For completeness we state that there is a generic spectral representation for the quark propagator satisfying the Schwinger-Dyson(SD) equation in QCD

$$S(p) = \int_0^\infty d\alpha \frac{\tilde{S}(\alpha)}{p^2 - \tilde{m}^2 - \alpha + i\epsilon} \quad (4)$$

We have introduced two important quantities[3] \tilde{m} and δ_n . \tilde{m} is the threshold mass *i.e.*, only for momenta $p^2 \geq \tilde{m}^2$ will the quark propagator have an imaginary part. It may in general be a pole if the quark is not confined or a branch cut if it is confined. Similarly δ_n is the relativistic analogue of the size(in units of mass) of the meson[2] of mass M_n . In particular the meson wave-function ϕ_n is a function of two complex variables, p^2 and \bar{p}^2 . Only for momenta $p^2 \geq \delta_n^2$ or $\bar{p}^2 \geq \delta_n^2$ the wave-function ϕ_n has singularities. The threshold mass \tilde{m} is in principle determined from the SD equation while the meson size δ_n and meson mass M_n are self-consistently determined from the BS equations. From eq.(2) it is easy to see that

$$0 \leq \delta_n \leq \min(\tilde{m}, \bar{\tilde{m}}) \quad (5)$$

The lower inequality follows from the fact that all amplitudes have no singularities in the space-like domain(here it amounts to saying the meson has a finite size) while the upper bound is self-evident from the BS equation, eq.(2) (\tilde{m} and $\bar{\tilde{m}}$ being the threshold masses of the quark and the anti-quark). A more precise estimate of δ_n demands the knowledge of the exact structure of the kernel in the BS equation. From the definition of δ_n (eq.(3)), we note that a bound quark in a meson has a threshold singularity at δ_n which is less than that of the unbound quark(eq.(5)).

Consider the amplitude of three quarks going to three quarks. We notice that there are contributions due to three different quark-quark scattering. One such contribution of interest is the one shown in Fig[2]. This involves one internal loop momentum k as explicitly shown in the Fig[2]. By choice we take q_i and r_i (with $r_1 = 0$) as independent momenta while the external momenta p_i and \bar{p}_i follow from momentum conservation at the vertices. To

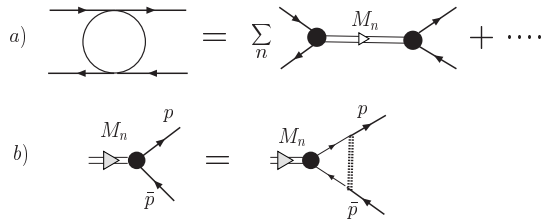


FIG. 1: (a)The Meson contributions to the quark anti-quark scattering kernel. (b) The Bethe-Salpeter equation for the Meson

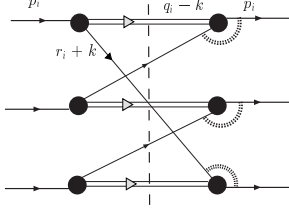


FIG. 2: Baryon pole(constituent quark picture).

avoid the ambiguity of double counting the quark propagator, we introduce the kernel used in the BS equation. Using spectral representation for the BS wavefunction(eq.(3)) this diagram is completely well defined and the loop momentum, k , integration can be done. We are interested in the singularity structure of this graph. We apply the Hadamard/Landau conditions[4] on all variables being integrated over, which includes y_i and β_i coming from each of the spectral representations of the meson vertices. To make the computation transparent first notice that we have to consider terms like

$$\Delta = y_i(k + r_i)^2 - (1 - y_i)p_i^2 - y_i(1 - y_i)M_i^2 - \delta_i^2 - \beta_i \quad (6)$$

This is equivalent to an effective mass term for the quark propagator which has momentum $(k + r_i)$. Landau equations demand that $\beta_i = 0$ and $y_i = 0, 1$ or $\frac{\partial \Delta}{\partial y_i} = 0$. Furthermore looking for the lowest effective mass with respect to p_i^2 i.e., $\frac{\partial \Delta}{\partial p_i^2} = 0$ we require $y_i = 1$. Consequently, we find $\Delta \approx (k + r_i)^2 - \delta_i^2$. To evaluate the nature of the singularity it is convenient to employ Cutkosky cuts[5] as shown by the dashed line in Fig[2]. With suitable conditions on $\tilde{\phi}(y, \beta)$ it yields six Dirac-delta functions which impose the following constraints on the momenta

$$(q_i^0 - k^0) = \sqrt{M_i^2 + (\vec{q}_i - \vec{k})^2} \quad (7)$$

$$(r_i^0 + k^0) = \sqrt{\delta_i^2 + (\vec{r}_i + \vec{k})^2} \quad (8)$$

Four of the Dirac-delta functions will eliminate the loop momentum k integration completely. One will be the overall pole condition on the total energy(eq.(10)) and the other Dirac-delta function restricts the external momenta. The loop momentum k integration also yields a self-consistency condition(Landau equation) namely the vanishing of the following four momenta

$$\sum_i \left(-a_i(q_i - k) + b_i(r_i + k) \right) = 0 \quad (9)$$

for some $a_i, b_i \geq 0$. The physical interpretation of this equation is that if the masses of all lines in the graph of Fig[2] are taken to be real, then the process depicted in this graph can occur in real space-time classically pro-

vided they obey this constraint[6]. The total energy is

$$E = \sum_{i=1}^3 (q_i^0 + r_i^0) = \sum_{i=1}^3 \sqrt{M_i^2 + (\vec{q}_i - \vec{k})^2} + \sqrt{\delta_i^2 + (\vec{r}_i + \vec{k})^2} \quad (10)$$

Minimising E , all spatial momenta vanish and we get the lowest baryon mass M_B ,

$$M_B = \sum_{i=1}^3 (M_i + \delta_i) \quad (11)$$

For this case, the condition in eq.(9) can be satisfied trivially. The mass of the lightest baryon, the proton, is in terms of the mass of the pion taken to be $M_\pi = 139 \text{ Mev}$ and the size of the pion, δ_π , inferred to be $174 \text{ Mev}(1 \text{ fm})$.

The picture that we have deduced from this analysis is precisely that of the constituent quark model. The interpretation of the graph(Fig[2]) is that there is a pole corresponding to a baryon state $|B\rangle$ and corresponds to the following matrix element,

$$\langle qq q | B \rangle \frac{1}{s - M_B^2 + i\epsilon} \langle B | qq q \rangle \quad (12)$$

where $s = E^2$ in the rest frame of the Baryon. Consequently we infer the overlap function $\langle B | qq q \rangle$ in terms of the explicit meson wave-functions. Furthermore for the lightest baryons the three quark momenta p_i are such that they are constrained to be $p_i^2 = (313 \text{ Mev})^2$. In the rest frame of the baryon these energies of the quarks decompose into masses M_π of the mesons and δ_π (bound quark mass in the meson) as shown by the cut in Fig[2].

In the theory Fig[2] is not the only pole contribution to the six quark Green's function. Indeed there are many others. Firstly this graph can repeat several times by itself or repeat with other quark propagations in between. All these also have singular Dirac-delta function contributions corresponding to the baryon pole. We envisage a systematic expansion wherein these contributions result in a finite renormalisation of the baryon mass M_B . Therefore our estimate for $\delta_\pi = 174 \text{ Mev}$ subsumes these contributions. In 't Hooft's $\frac{1}{N}$ expansion[7] where N is the number of colors, we know that every additional meson function implies a factor of $\frac{1}{\sqrt{N}}$. Some of these finite renormalisations are controlled in the $\frac{1}{N}$ expansion. Indeed these contributions makes the evaluation of masses of excited baryons tedious and also detailed knowledge of meson wave-functions becomes necessary.

In addition there are other corrections which allows us to infer Feynman's parton model. For example, using the same technique we can look at Green's functions where the three quark state goes to three quarks and several other gluons and quark anti-quark pairs as shown in Fig[3]. All these graphs also have the same pole contribution as shown by the cut in Fig[3] and indeed this graph

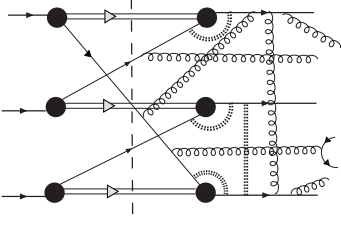


FIG. 3: Baryon pole(Parton picture).

corresponds to

$$\langle qq\bar{q}|B\rangle \frac{1}{s - M_B^2 + i\epsilon} \langle B|qq\bar{q}, gg(etc)\rangle \quad (13)$$

where g stands for gluons. From this we infer $\langle B|qq\bar{q}, gg(etc)\rangle$ matrix elements explicitly. That is the baryon state has a finite probability amplitude to produce various other degrees of freedom in addition to the valence quarks.

$$\langle B| = \langle B|qq\bar{q}\rangle\langle qq\bar{q}| + \langle B|qq\bar{q}gg\rangle\langle qq\bar{q}gg| + \dots \quad (14)$$

From our analysis we see that each term on the the *r.h.s* of the above equation has the same mass eigenvalue M_B as dictated from Fig[3]. It should be noticed that the constituent quark picture of Fig[2] is disturbed because these constituents can also exchange energy as shown in the Fig[3] leading to parton distribution functions. We also note in the conventional parton picture, quarks with momenta \bar{p}_i (Fig[2]) emit gluons but we find in general there are quarks with lower energy, of the order of δ_i which also emit gluons as shown in Fig[3]. Although formally all these are precisely given in terms of meson wave-functions and quark and gluon propagators, a systematic convergent expansion is an open problem.

Stability of the baryon state would require that $M_B \leq 3\tilde{m}$. We have no *a priori* estimate of \tilde{m} in QCD but a self-consistent picture to be presented later shows that $\tilde{m} \leq 591\text{Mev}$. In a model theory such as σ QCD[8] it has been estimated[3] for u, d, s quarks to be about 550Mev .

Excited baryons can come about because of excited mesons in the intermediate states of Fig[2]. Note that excited mesons such as ρ, ω etc are necessarily unstable because they do decay into 2π or 3π states. Consequently the meson pole is in the complex energy domain. Our analysis of the Green's functions can be carried out even in the complex plane consistent with causality (*i.e.*, with Feynman $i\epsilon$ prescription) namely positive energy pole has negative imaginary part and *vice versa*. The external momenta p_i and \bar{p}_i then have to be complex. As a consequence the width of the baryon is necessarily greater than the widths of these intermediary mesons. For example if we consider intermediate mesons to be ρ and π with masses M_ρ and $2M_\pi$, we

get $M_B = 2(M_\pi + \delta_\pi) + (M_\rho + \delta_\rho) \approx 1474\text{Mev}$ for $\delta_\rho \approx \frac{\delta_\pi}{2}$ (since the size of the ρ meson has to be larger than that of the pion). This matches with the Roper resonance and we would also find that the widths obey $\Gamma_{\text{Roper}} > \Gamma_\rho \approx 150\text{Mev}$, which is consistent with phenomenology. As we see in this example predicting higher states demands knowing meson properties, M_n and δ_n . The latter being unavailable we have to develop theoretical models. As another example, consider the Λ particle which is made up of u, d, s quarks. The intermediate mesons in Fig[2] have to be two π and K^0 . From the masses of these mesons we get the mass of the Λ baryon, $M_\Lambda = 1.2\text{Gev}$ for $\delta_K \approx \frac{\delta_\pi}{2}$.

Our method of estimating the spectrum of baryons although unconventional is consistent with quantum field theory. It is also consistent with the Hamiltonian eigenvalue problem. The complication is that the eigenvalue problem is stated in the basis of infinite Fock states. It is interesting to note that the same eigenvalue problem (eq.(14)) can be solved by using Green's functions and their singularities. We can explicitly get the mass eigenvalues and baryon states in terms of meson states. Now, it has to be noted that if we take a system which is essentially non-relativistic, *i.e.*, the threshold masses are large compared to the binding energy, b_i , then the mass of the 2-body meson bound state is given by, $M_i \approx 2\tilde{m} - b_i$ and $\delta_i \approx a\tilde{m}$ where a is some interaction coupling constant. Consequently we will estimate the baryon mass from Fig[2] to be $3(2\tilde{m} + a\tilde{m} - b_i)$ which is much greater than $3\tilde{m}$ (mass of three free quark states) implying that the system is unstable. This shows our procedure is inapplicable for all heavy quark systems. Similar considerations on charm and u, d quark systems also results in instability for the baryon bound state.

Exact chiral symmetry is another limit where we can draw conclusions. This as discussed in [3] is a singular limit. Due to Goldstone's theorem we have an exact relationship between the pion and the quark propagator [9] namely $\phi_\pi(p) \propto \gamma_5 \text{Tr} S(p)$ showing that $\delta_\pi = \tilde{m}$ and also $M_\pi = 0$. Consequently the mass of the baryon is $M_B = 3\tilde{m}$ implying that the baryon is almost unstable. Hence in the constituent quark model estimate, the baryon cannot be bound. However there are finite renormalisation effects alluded to earlier. These effects can give rise to a weakly bound baryon state.

Now consider the recently discovered[10] pentaquark candidate which has been discussed in the literature[11]. This is a $udud\bar{s}$ color singlet system with mass 1540Mev . In our picture the graph and cut shown in Fig[4] gives the lowest pentaquark possible. The mass of the pentaquark θ is therefore

$$M_\theta = 3(M_\pi + \delta_\pi) + M_K + M_\pi = M_P + M_K + M_\pi = 1571\text{Mev} \quad (15)$$

where M_P is the mass of the proton or neutron and M_K is the mass of the K meson. Interestingly this prediction

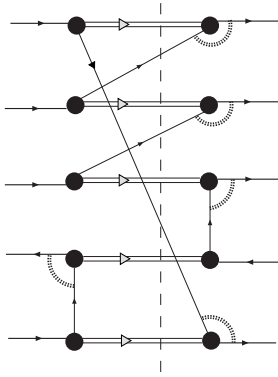


FIG. 4: Pentaquark pole.

does not explicitly depend on δ_π if the mass of the proton is given. Fig[4] shows that the spin of this state is $\frac{1}{2}$ as in the case of the proton. As far as the width is concerned, since it is made up of only stable mesons, it is expected to be narrow.

We may look at other pentaquarks such as $udud\bar{u}$, our prediction for its mass will be $M_P + 2M_\pi = 1217\text{Mev}$ and spin $\frac{1}{2}$, which is close to the mass of the Δ baryon. This leads us to suspect that in the standard Δ resonance which has a width of about 120Mev there exists a pentaquark resonance as well. The standard Δ resonance is the uuu state which in our picture comes about from symmetry considerations in the same way as in the constituent quark model. In terms of graphs, there are actually two graphs of the Fig[2] type and the cancellations between them yields a higher mass to the uuu system than an uud system. Similarly the masses of heptaquark states such as $udus\bar{d}s\bar{u}$ can also be predicted to be $M_P + 2M_K + 2M_\pi = 2313\text{Mev}$ and this resonance will be a little broader since it has many decay channels.

In our considerations here, the important infrared properties of the quarks and mesons that are relevant to the description of baryons are the threshold masses, \tilde{m} , meson masses, M_n and sizes, δ_n . We have experimental knowledge of M_n and to a lesser extent of δ_n inferred from form factors. In principle \tilde{m} and δ_n can be inferred from Lattice simulation of QCD. Another attempt is to model QCD $q\bar{q}$ scattering kernel in the BS equation(Fig[1b]) in a reliable manner. σQCD [8] is one such approach where the kernel is taken to be purely a one gluon exchange and a singular ' $\frac{\sigma}{q^2}$ ' propagator. Here q is the exchanged momentum between the quark and the anti-quark. This was analysed in the large N expansion with g^2N (g is the gauge coupling constant) also small. In this theory which is asymptotically free, we find spontaneous symmetry breakdown of chiral symmetry and PCAC[2, 3]. The threshold mass \tilde{m} of u, d, s quarks is numerically estimated to be 550Mev and δ_π can be estimated.

In the above discussion even if the quark is not confined

i.e., the quark propagator has a pole at \tilde{m} , as long as the inequalities alluded to in the paper are satisfied the results remain true. Color non-singlets such as diquark systems can also be investigated in our picture. The graph of this system is that of Fig[2] but with the middle quark line removed. In 4 dimensions the loop momentum is uniquely fixed and there is no residual Dirac-delta function to restrict the total energy of the Green's function. This implies the diquark system has no pole but only a branch cut singularity. This may have important consequences in scattering processes. Similarly if we take a four quark system, they can be bound like the three quark system with a mass $4(m_\pi + \delta_\pi)$ even though they can never be a color singlet. Looking at non-singlets one can consider a $qq\bar{q}$ system which has the same color quantum numbers as a fundamental quark. This yields the same graph as in Fig[4] but without two quarks. The estimate of the mass of this state being $3m_\pi + \delta_\pi = 591\text{Mev}$ and is necessarily a branch cut. Consequently consistency of our picture demands that the threshold mass be,

$$\tilde{m} \leq 591\text{Mev} \quad (16)$$

Color singlet criterion has played very little role in our analysis, if at all these considerations have to come from other theoretical aspects of QCD. It is assumed that mesons are color singlets. It should be borne in mind that production of these multi-quark states becomes harder because these have to obey the criteria of eq.(9) in the real phase space.

* email(corresponding author): ramesha@imsc.res.in

† email: sant@imsc.res.in

- [1] S. Deser, W. G. Gilbert, E. C. G. Sudharshan, Phys.Rev.**115** 731(1959); M.Ida, Prog.Theor.Phys. 2345(1960).
- [2] R.Anishetty, S.K.Kudtarkar, Phys.Lett.**B574**, 175 (2003).
- [3] R.Anishetty, S.K. Kudtarkar, Phys.Lett.**B569**, 47 (2003).
- [4] R.J.Eden *et al.*, *The Analytic S-Matrix*, CUP, 1966.
- [5] R.E.Cutkosky, J.Math.Phys.**1**, 429(1960); G.'tHooft, M.J.Veltman, *Diagrammar*, cernrep/1973-009, (1973).
- [6] S.Coleman, R.E Norton, Il.Nuov.Cim **38N1**, 438 (1965).
- [7] G.'tHooft, Nucl. Phys. B **72** 461 (1974).
- [8] Ramesh Anishetty, hep-ph/9804204.
- [9] R.Jackiw, K.Johnson, Phys.Rev.D **8**, 2386 (1973).
- [10] T.Nakano, *et al.*, (*LEPS*), Phys.Rev.Lett.**91**, 012002(2003); V.V.Barmin, *et al.*, (*DIANA*), Yad.Fiz.**66**, 1763(2003); S.Stepanyan, *et al.*, (*CLAS*), Phys.Rev.Lett.**91**, 252001(2003); A.Airapetian, *et al.*, (*HERMES*), Phys.Lett.B **585**, 213(2004).
- [11] R.L. Jaffe, F. Wilczek, Phys.Rev.Lett.**91**, 232003(2003).
- [12] D. Binosi, L. Theussl, (*JAXODRAW*), hep-ph/0309015.